

# Solving the Dynamic Vehicle Routing Problem Under Traffic Congestion

Gitae Kim, Yew Soon Ong, Taesu Cheong, and Puay Siew Tan

**Abstract**—This paper proposes a dynamic vehicle routing problem (DVRP) model with nonstationary stochastic travel times under traffic congestion. Depending on the traffic conditions, the travel time between two nodes, particularly in a city, may not be proportional to distance and changes both dynamically and stochastically over time. Considering this environment, we propose a Markov decision process model to solve this problem and adopt a rollout-based approach to the solution, using approximate dynamic programming to avoid the curse of dimensionality. We also investigate how to estimate the probability distribution of travel times of arcs which, reflecting reality, are considered to consist of multiple road segments. Experiments are conducted using a real-world problem faced by Singapore logistics/delivery company and authentic road traffic information.

**Index Terms**—Dynamic vehicle routing problem, approximate dynamic programming, uncertain travel times, rollout algorithm.

## I. INTRODUCTION

AS urbanization has increased rapidly over the last few decades, city logistics and last-mile logistics have become active fields of research with applications in industry as well as academia. Thus, the dynamic vehicle routing problem (DVRP) in urban contexts has received more attention in the literature as intelligent transportation systems (ITS) have rapidly improved. A major complication of the DVRP is the dynamics of travel times caused by fluctuating traffic conditions, especially traffic congestion. Traffic congestion wastes enormous quantities of

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money, time, and fuel in urban areas [11]. Thus, the development of an efficient strategy for vehicle routing under traffic congestion is essential. This paper proposes a DVRP model with non-stationary stochastic travel times for a distribution system consisting of a depot and multiple customers. We also investigate the details of estimating the travel time distribution.

Travel times have been assumed to be time-dependent, stochastic (random), or both time-dependent and stochastic. Cooke and Halsey [9] introduced a model featuring a time-dependent network and Frank [16] introduced one featuring stochastic travel times. However, Hall [23] first considered both time-dependent and stochastic travel times together in a routing decision model and proposed adaptive decision rules for making route choices. This topic has been of keen interest in the field of intelligent transportation systems (ITS). However, when it comes to analyzing both time-dependent and stochastic travel times in a transportation network for routing or touring decision problems, finding an optimal solution remains a challenge. For instance, the standard shortest path algorithm such as Dijkstra's algorithm may fail to find an optimal solution in a transportation network where arc travel times are both time-dependent and stochastic [23]. Likewise, the vehicle routing problems with time-dependent stochastic travel times aim to find the best route policy rather than the optimal route.

Urbanization causes customers to increasingly require on-time delivery, for which, in urban areas, limited road capacity and traffic congestion are major obstacles. Accordingly, the cost of congestion has rapidly increased in metropolitan areas and its unpredictability has forced logistics providers to allow sufficient buffer times in order to satisfy on-time delivery performance targets [20]. Although many studies on the DVRP with either time-dependent or stochastic travel times have been conducted in the literature, to the best of our knowledge, there is a lack of studies on the DVRP considering both time-dependent and stochastic travel times simultaneously. We remark that, in practice, travel times may change not only time-dependently but also randomly as the status of the traffic congestion evolves over time. In the DVRP, the decision maker or vehicle driver determines the next immediate destination along a tour at each decision epoch based on visit history and traffic congestion information. Due to the nature of the problem which involves sequential decision-making and the stochastic properties of changes in traffic conditions, a stochastic dynamic programming model, specifically a Markov decision process, is employed. Furthermore, most vehicle routing problems assume the arc between two nodes to be a single unit, but, in reality, it is made up of a sequence of road segments joining the two

end locations, and each road segment may have different travel time distributions. Thus, when estimating the travel time distributions of each arc, we take into account the characteristics of each road segment.

In this paper, we consider a DVRP with time-dependent and stochastic travel times under traffic congestion and formulate the problem with the Markov decision process (MDP) model. The consideration of this problem in this paper is motivated by the problem of a Singapore delivery company, the delivery network of which consists of a single depot and multiple customers. Customer demands are known, but travel times between customer locations are time-dependent and stochastic mainly due to traffic congestion. The dynamics of travel times require more states than customers since the number of arcs is larger than the number of nodes. In creating the solution, we employ a rollout policy, a neuro-dynamic programming algorithm, to help resolve the curse of dimensionality. A rollout algorithm is an approximate dynamic programming (ADP) method based on policy iteration and a look-ahead procedure, assuming a given base policy (heuristic). Focusing on the decision policy of a vehicle for a given route, we assume static clustering and treat the customer groups of the delivery company as clusters for the vehicles.

Thus, we here propose a dynamic programming model for a DVRP in a transportation network where travel times are time-dependent and stochastic, with a rollout policy based approach to the solution. We also suggest a realistic estimation procedure for arc travel time distributions using the travel time distributions of every road segment on the arc. This paper proposes a MDP model for the DVRP with time-dependent stochastic travel times. A transportation system based on a delivery company in Singapore and its related traffic information is used to formulate the model. A single depot and multiple customers are located on the map. The distance between two locations is derived by the total distance of the shortest path, the optimal directions based on the map. The arc travel time distribution is estimated by combining the time distributions of the road segments connecting the two ends of the arc to represent the situation more realistically. We also adopt a one-step rollout policy to deal with the curse of dimensionality and achieve useful solutions within a reasonable time frame.

The remainder of the paper is as follows. Section II briefly reviews some relevant literature. Section III presents a description of the problem and the proposed Markov decision process model. Section IV presents the procedures for estimating the distributions of traffic congestion and travel times. Next, the rollout approach as a solution method is proposed in Section V. Section VI gives numerical examples using the case study of a delivery company in Singapore. Finally, Section VII discusses conclusions and possible future research.

## II. LITERATURE REVIEW

In vehicle routing, there are various dynamics such as demand, travel time, service time, time windows, and vehicle/driver availability. Travel time is an important source of dynamics, particularly in urban areas [15]. Dynamic travel time,

known as time-dependent travel time, has two types: deterministic and stochastic with respect to the source of traffic information. Specifically, the stochastic dynamic travel time uses historical traffic data to estimate the probabilistic distribution of travel times.

Cooke and Halsey [9] introduced the problem with time-dependent travel time. The paper assumed that arc travel time varies as a function of the starting time from the start node. They modeled the shortest path problem using Bellman's iteration scheme. With the advancement in information technology (IT) and the demand for ever more intelligent transportation systems (ITS), time-dependent travel time has become of greater importance in the recent years. Koutsopoulos and Xu [26] pointed out that as the distance on an arc starting from the current node increases or the variability of travel time on the arc increases, real time traffic information becomes less useful. For such cases, they suggested that traffic information derived from historical data is better than real time traffic information. Intelligent transportation systems use real time traffic information to address time-dependent travel times [11], [22], [29]. Taniguchi and Shimamoto [40] used a simulation method to estimate congestion arcs and travel times. Lee *et al.* [27] employed a robust optimization approach to solve the uncertainty of travel time and demand. Recently, Wang *et al.* [43] proposed a method which directly collects real time traffic information using individual probe vehicles.

The travel time distribution has been assumed in various ways. For example, Figliozzi [13], [14] expressed the travel time on links as a function of the departure time which is calculated by constant or time dependent travel speed. Schilde *et al.* [37] explored a dynamic dial-a-ride problem with stochastic time dependent travel speed. From the history data, they estimated the distribution of transportation requests by Poisson distribution, and the time between the request and the latest vehicle arrival time by Gamma distribution derived from the distribution of travel speeds and travel times. Tas *et al.* [41], [42] assumed travel times as independent identical Gamma distributions in VRP with soft time windows and service costs. To reduce the efforts of discretizing the continuous distribution for the travel time, Gao and Huang [18] and Nielsen *et al.* [31] assumed the travel time as an independent integer-valued discrete random variable. We observe in literature that travel times are also commonly assumed to follow normal distributions. Figliozzi [12] assumed that the travel time of a commercial vehicle in urban area follows normal distribution. Guner *et al.* [20] proposed a dynamic stochastic routing model under recurrent or non-recurrent congestions and assumed the link travel time follows normal distribution. Fu and Rilett [17] modeled the dynamic stochastic shortest path problem as a stochastic process and assumed the travel time is normally distributed. Nakamura *et al.* [30] adopted the same distribution as in Fu and Rilett [17] for travel times in the dynamic stochastic vehicle routing and scheduling problem. Chang *et al.* [6] assumed normal distribution for the travel times in the stochastic dynamic traveling salesman problem and conducted the standard Kolmogorov-Smirnov test to support the assumption of normal distribution, which was also confirmed by simulation results in Chang *et al.* [6].

Hall [23] investigated a transportation network where the arc travel time is time-dependent as well as stochastic. In this case, the standard shortest path algorithm may be insufficient to find the optimal path [17]. Kim *et al.* [25] proposed a method to derive the travel time distribution for a non-stationary stochastic shortest path problem. Guner *et al.* [20] adopted the Gaussian mixture model to estimate arc congestion states given real time velocity information. Azadian *et al.* [1] extended the model in [20] for the time-dependent and stochastic shortest path problem to an air-cargo flight routing problem.

However, interestingly, while there are many studies on vehicle routing problems which assume time-dependent travel times only (for example, [9], [11], [22], [26], [27], [29], [40], [43]), to the best of our knowledge, there are few on vehicle routing problems with time-dependent and stochastic travel times [24]. Of course, the literature does include research studying shortest path problems for both time-dependent and stochastic travel times [15], [17], [20], [23], [25].

Dynamic programming (DP), particularly MDP, is suitable to DVRP because it makes sequential dynamic decisions over a time horizon with uncertainty. Unlike the shortest path problem, the DVRP with travel time dynamics and fixed demands has a constant number of stages which correspond to the number of customers each vehicle visits. This property encourages one to apply MDP to the problem. However, due to the *curse of dimensionality*, the approximate dynamic programming (ADP) approach has been used in practice [36]. The neuro-dynamic programming method of rollout algorithms is a novel approach to ADP [4], [5]. DVRP studies have included the use of rollout algorithms and neuro-dynamic programs [19], [21], [32], [38]. Secomandi [39] analyzed the performance of the rollout policy both theoretically and computationally for sequencing problems such as the traveling salesman problem with stochastic travel times (TSPST) and vehicle routing problems with stochastic demand (VRPSD). They showed that the rollout algorithm guarantees the improvement of the quality of any feasible solution to a given sequencing problem and that it is suitable for time-dependent and stochastic problems. The rollout algorithm can be applied to a wide variety of domains, including not only machine maintenance and repair, stochastic scheduling, facility location, computer code scheduling and revenue management [39], but also production scheduling in pharmaceutical manufacturing [8] and stochastic resource allocation [10]. However, the DVRP with time-dependent and stochastic travel times has received little attention in the literature.

### III. PROBLEM DESCRIPTION AND FORMULATION

We consider a DVRP with non-stationary stochastic travel times. In a distribution system, vehicles deliver goods from a depot to customers. Customers are assumed determined before the delivery, and all of them need to be served. The traffic congestion status of the distribution network is assumed to evolve over time according to a stochastic process, and this evolution characterizes non-stationary stochastic travel times. Consider a complete graph  $G = (\mathcal{N}, \mathcal{A})$  where  $\mathcal{N} = \{0, 1, 2, \dots, l\}$  is the set of nodes (0: node of depot,  $1 \sim l$ : nodes of customer locations) and  $\mathcal{A} = \{a \equiv (n, n') | n, n' \in \mathcal{N}, n \neq n'\}$  is a set of

arcs. An arc  $(n, n') \in \mathcal{A}$  is said to be *observable* if real-time traffic congestion information can be captured on  $(n, n')$ , and the set of observable arcs is denoted by  $\mathcal{A}' \subseteq \mathcal{A}$ . Let  $s_a(t)$  denote the traffic congestion state for an arc  $a \in \mathcal{A}'$  in the network at time  $t$  and  $\vec{s}_t = \{s_1(t), s_2(t), \dots, s_{|\mathcal{A}'|}(t)\}$  be the (random) network congestion status vector at time  $t$  where time  $t$  is assumed to be discrete. We denote  $\Gamma = \{1, 2, \dots, \tau\}$  to be the set of vehicles with capacity  $Q$ . The customer set is exclusively partitioned into the set of clusters as  $\mathcal{N}' = \mathcal{N} \setminus \{0\} = \mathcal{N}_1 \cup \mathcal{N}_2 \cup \dots \cup \mathcal{N}_K$  where  $\mathcal{N}_i \cap \mathcal{N}_j \equiv \emptyset$ ,  $i \neq j$  and each vehicle is assigned to each cluster  $\mathcal{N}_j$ ,  $j = 1, 2, \dots, K$ . We further assume that the number of vehicles is enough to cover all the clusters (i.e.,  $K < \tau$ ). Customer clusters are assumed deterministic, which means that the clusters do not change with time. We assume that the arc traffic congestion state  $\{s_a(t)\}$  for each arc  $a \in \mathcal{A}'$  evolves over time  $t$  according to a non-stationary Markov chain, and that the congestion state of any given arc is independent of the congestion states of the others. The realization of  $s_a(t)$  has a discrete value which corresponds to a certain level of congestion, such as  $s_a(t) \in \{0, 1\}$  (0: uncongested, 1: congested) or  $s_a(t) \in \{0, 1, 2\}$  (0: uncongested, 1: mildly congested, 2: heavily congested). We denote  $p_{ss'}^a(t) = P(s_a(t+1) = s' | s_a(t) = s)$ ,  $a \in \mathcal{A}'$  as a one-step transition probability of traffic congestion states for an arc  $a$  that is associated with the transition from state  $s$  at time  $t$  to state  $s'$  at time  $t+1$ , and the transition matrix  $p_{t,t+1}^a$  is a  $|s_a(t)| \times |s_a(t)|$  matrix with the elements  $p_{ss'}^a(t)$ . The probability of traffic congestion states on the arc is estimated by using the average vehicle speed on the arc, as discussed in Section IV. Assuming that arc congestion states in  $\mathcal{A}'$  are independent of each other, the probability of a state transition occurring from  $\vec{s}$  at time  $t$  to  $\vec{s}'$  at time  $t+n$ ,  $P(\vec{s}' | t, \vec{s}, t+n)$ , is

$$P(\vec{s}' | t, \vec{s}, t+n) = \prod_{a=1}^{|\mathcal{A}'|} P(s_a(t+n) = \langle \vec{s}' \rangle_a | s_a(t) = \langle \vec{s} \rangle_a)$$

where  $\langle \vec{s} \rangle_a$  is the  $a$ -th element of  $\vec{s}$ .

By Kolmogorov's equations, the transition matrix for the multi-stage case is as follows [25]:

$$P_{t,t+n}^a = [P_{t,t+1}^a] \times [P_{t+1,t+2}^a] \times \dots \times [P_{t+n,t+n+1}^a].$$

Now, we consider the travel time between two nodes. Denote  $P(x' | n, t, \vec{s}_t, n')$  to be the probability that the travel time from node  $n$  at time  $t$  to node  $n'$  with traffic congestion state  $\vec{s}_t$  at time  $t$  is  $x'$ . We assume that the travel times for each arc between two nodes follow a normal distribution and they are discretized in the model. Each arc's travel time distribution is derived by the convolution of distributions of the road segments between the two nodes which make up the arc, as discussed in the next section.

Let  $c(n, t, \vec{s}_t, n', t')$  be the travel cost function from node  $n$  at time  $t$  to node  $n'$  at time  $t'$  when the vehicle leaves from node  $n$  with traffic congestion state  $\vec{s}_t$  at time  $t$ . We also let  $g(n, t, \vec{s}_t, n')$  be the expected travel cost function from node  $n$  at time  $t$  to node  $n'$  when the vehicle leaves from node  $n$  with traffic state  $\vec{s}_t$ . Like the travel times, we assume that the cost of travel between two nodes is not affected by the traffic states of the other arcs. Although the travel cost may include

driver's labor, fuel and any operation costs, we assume that it is proportional to the travel time (i.e.,  $c(n, t, \vec{s}_t, n', t') \propto (t' - t)$ ). Thus, the expected cost of the travel time between two nodes is defined as:

$$g(n, t, \vec{s}_t, n') = \sum_{x'} P(x'|n, t, \vec{s}_t, n') c(n, t, \vec{s}_t, n', t + x').$$

Given a set of customer clusters, the concomitant problem can be decomposed into single-vehicle problems. For each vehicle  $k$ , we can formulate a Markov decision process model. For simplicity of explanation of the proposed model, we will hereafter consider a single vehicle routing model, which can be identically applied to the problem of each vehicle.

The state of the system is defined as  $\Omega(n, t, \vec{s}_t, M_t)$  where  $M_t (\subseteq \mathcal{N}_k)$  is the set of unfulfilled customers at time  $t$  and excludes the customer at the current node  $n$ . The state  $\Omega(n, t, \vec{s}_t, M_t)$  denotes that the vehicle has just served customer  $n$  and is still located at node  $n$  at time  $t$  with traffic state  $\vec{s}_t$  and has to visit all customers in the set  $M_t$ . Define  $\Psi = \{\pi(\Omega_0), \pi(\Omega_1), \dots, \pi(\Omega_{|N_k|})\}$  as the set of decision rules or policies. The decision is 'where to go next' for each vehicle at the current state. For instance,  $\pi(\Omega_t) \in R_t (\subseteq M_t)$  where  $R_t$  is the set of reachable nodes from current node at time  $t$ . The objective of the model is to minimize the total expected travel cost accrued by visiting all of the nodes in  $\mathcal{N}'$ . The expected travel cost when the current node is  $n$  at time  $t$ , the traffic congestion state is  $\vec{s}_t$ , and the next node to visit is  $n' \in R_t$  is defined as

$$\omega(n, t, \vec{s}_t, n') = g(n, t, \vec{s}_t, n') + \sum_{x'} P(x'|n, t, \vec{s}_t, n') \times \sum_{\forall \vec{s}'_t} P(\vec{s}'_t|t, \vec{s}_t, t + x') v^*(n', t + x', \vec{s}'_t, M_t \setminus \{n'\}) \quad (1)$$

where  $v^*(n', t', \vec{s}'_t, M_t \setminus \{n'\})$ , known as the cost-go-function, is the minimum of the expected travel cost functions accrued starting from node  $n'$  at time  $t'$  with traffic congestion state  $\vec{s}'_t$  until the remaining nodes are fulfilled. Thus, the optimality equation becomes

$$v^*(n, t, \vec{s}_t, M_t) = \min_{n' \in R_t} \{\omega(n, t, \vec{s}_t, n')\} \quad (2)$$

with boundary condition  $v^*(n, t, \vec{s}_t, \phi) = g(n, t, \vec{s}_t, 0)$  for all  $n \in \mathcal{N}'$ . We remark that the optimality equations (1) and (2) affirm that the proposed model takes the non-stationary or time-variant aspects into account (refer to [7] for the stochastic TSP with stationary Markov chains). Then, the optimal policy under the current state at time  $t$  is determined as follows:

$$\pi^*(\Omega_t) \equiv \arg \min_{n' \in R_t} \{\omega(n, t, \vec{s}_t, n')\}.$$

To solve the concomitant problem above, we need to calculate Equation (1), and this notably includes calculating the expected cost-to-go function recursively for all states. However, as the size of nodes (i.e.,  $|\mathcal{N}'|$ ) increases, finding all the optimal values of the cost-to-go functions becomes computationally intractable. Therefore, we employ an approximate dynamic

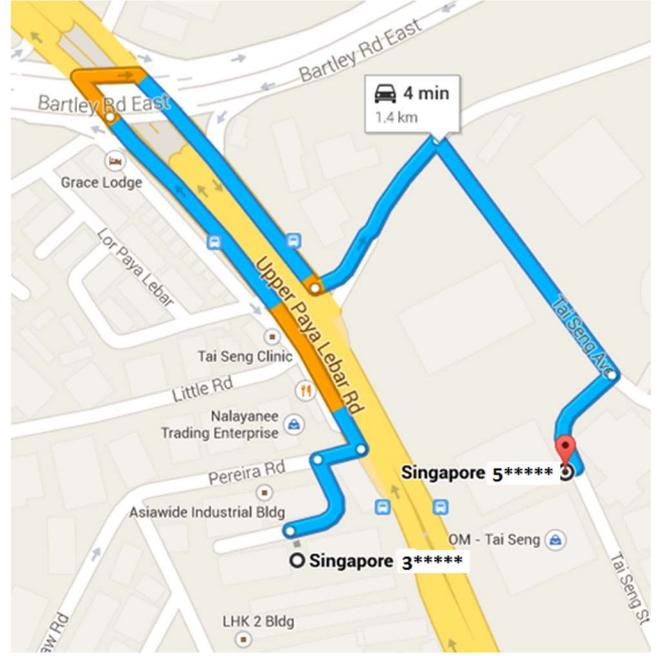


Fig. 1. Illustration of an arc consisting of multiple road segments.

programming approach based on the rollout policy which, although it produces a sub-optimal solution, significantly reduces the computational burden.

#### IV. ESTIMATION OF TRAFFIC CONGESTION AND TRAVEL TIME DISTRIBUTION

We now discuss a methodology for deriving the dynamics of traffic congestion states on each arc in a network  $G$ , and estimating the probability distributions of the travel times on the arc. In most relevant studies, an arc between two nodes has been considered as a single unit (for example, [7]). However, in reality, an arc may consist of multiple road segments. Let us consider an example from the company data under consideration. Fig. 1 shows two locations (customers) on the map. When a vehicle travels from node 1 with zip code 3\*\*\*\*\* to node 2 with zip code 5\*\*\*\*\*, there are many alternative paths. However, when the company's truck drivers look for a way to get from node 1 to node 2, they rely on GPS navigation systems to guide them, which generally simply choose the shortest paths between nodes. Fig. 1 shows the directions given by Google Maps. The details of the directions presented in Fig. 1 are as follows:

- 1) Head east toward Pereira Rd
- 2) Turn right onto Pereira Rd
- 3) Turn left onto Upper Paya Lebar Rd
- 4) Make a U-turn at Bartley Rd East
- 5) Turn left onto Kim Chuan Rd
- 6) Turn right onto Tai Seng Ave
- 7) Take the 1st right onto Tai Seng St

Thus, the arc connecting the two nodes in network  $G$  is physically made up of six road segments: Pereira Rd, Upper Paya Lebar Rd, Bartley Rd East, Kim Chuan Rd, Tai Seng Ave, and Tai Seng St. While the Euclidean distance between the two nodes in Fig. 1 is 0.25 km, the distance of the path in Fig. 1 is

TABLE I  
DISTANCES OF ROAD SEGMENTS BETWEEN NODE 1 AND NODE 2

Road Segments	Distance (m)	Percentage
Pereira Rd	137	9%
Upper Paya Lebar Rd	450	31%
Bartley Rd East	300	21%
Kim Chun Rd	180	12%
Tai Seng Ave	270	19%
Tai Seng St	110	8%
<b>Total</b>	<b>1,447</b>	<b>100%</b>

1.4 km, more than 5 times longer. In this regard, for the city area transportation network in network  $G$ , using the Euclidean distance as the distance of an arc is inappropriate. Instead, we should use the distance of the shortest path connecting the two nodes. Table I shows the distances of the six road segments between node 1 and node 2.

Likewise, when estimating the travel time of an arc in  $G$ , we should consider the travel times of the road segments constituting the path. That is, the travel time distribution for each arc is estimated considering the travel time distributions of the road segments corresponding to that arc, and the travel time probability distributions for each arc derived in this way are then used in the Markov decision model presented earlier.

#### A. Dynamics of Traffic Congestion

In this section, we discuss how to estimate the traffic congestion state transition probabilities for each arc in  $\mathcal{A}'$ . The traffic information is collected by sensors on the roads, and those sensors store the average velocity of vehicles on the road over time. That information is used to obtain the average traffic velocity on the road. In order to estimate the dynamics of traffic congestion states over time, we assume that the average speed of each road segment follows a non-stationary normal distribution. The distribution of the velocity on each road segment is used to estimate the distribution of congestion states [20], [25]. As discussed in Section III, the transition probability of a traffic congestion state from state  $s$  to  $s'$  at time  $t$  is  $p_{ss'}^a(t) = P(s_a(t+1) = s' | s_a(t) = s)$ ,  $a \in \mathcal{A}'$ . A traffic congestion state  $s_a(t)$  can be defined based on the range of the velocity [20], [25]. For instance,  $P(s_a(t) = s) = P(v_l \leq v_a(t) \leq v_u)$  where  $v_a(t)$  is the average velocity at time  $t$  on arc  $a$  and  $[v_l, v_u]$  is the range of velocities associated with state  $s$ . The range of velocities can be divided into traffic congestion levels in many different ways: for example, two congestion levels (*congested* or *uncongested*), and three congestion levels (*heavily congested*, *mildly congested* or *uncongested*), and so on.

Let  $V(t)$  be the random variable of the velocity for arc  $a$  between two customers at time  $t$ . Let us assume that there are  $m$  road segments within arc  $a$ . Denote the random variables for the velocities of  $m$  road segments as  $V_1(t), V_2(t), \dots, V_m(t)$  respectively, which are assumed to be independent of each other and follow normal distributions (see Appendix for details). The random variable for the velocity of arc  $a$  can then be described

by linear combination of the random variables of the road segments:

$$V(t) = \varepsilon_1 V_1(t) + \varepsilon_2 V_2(t) + \dots + \varepsilon_m V_m(t) \quad (3)$$

where the coefficient  $\varepsilon_i$  in (3) is the proportion of the distance of road segment  $i$  of the total distance of arc  $a$ . For the example of Fig. 1, the random variable of the velocity of the arc between nodes 1 and 2 can be expressed by  $V(t) = 0.09V_1(t) + 0.31V_2(t) + 0.21V_3(t) + 0.12V_4(t) + 0.19V_5(t) + 0.08V_6(t)$ . Since  $V(t)$  is a linear combination of normal random variables, it also follows a normal distribution. The mean and variance of  $V(t)$  are derived as follows:

$$\begin{aligned} \mu_{V(t)} &= E[V(t)] = \varepsilon_1 E[V_1(t)] + \varepsilon_2 E[V_2(t)] + \dots + \varepsilon_m E[V_m(t)] \\ \sigma_{V(t)}^2 &= \text{Var}[V(t)] = \varepsilon_1^2 \text{Var}[V_1(t)] + \varepsilon_2^2 \text{Var}[V_2(t)] \\ &\quad + \dots + \varepsilon_m^2 \text{Var}[V_m(t)]. \end{aligned}$$

Therefore, the velocity of arc  $a$  follows a normal distribution of  $N(\mu_{V(t)}, \sigma_{V(t)}^2)$ . Based on the probability distribution of  $V(t)$ , we can estimate the one-step transition probability of the congestion states as follows:

$$\begin{aligned} P(s_a(t+1) = s' | s_a(t) = s) \\ = \frac{P(v_l \leq V(t) \leq v_u \cap v_{l'} \leq V(t+1) \leq v_{u'})}{P(v_l \leq V(t) \leq v_u)} \quad (4) \end{aligned}$$

where  $[v_{l'}, v_{u'}]$  is a range of velocities associated with the state  $s'$ . The numerator in Equation (4) is the joint velocity distribution of consecutive time periods and can be approximated by a bivariate normal distribution [25].

#### B. Travel Time Estimation

Another important constituent of the proposed approach is the estimation of the travel time distributions of each arc. The distance between two nodes is constant and obtained by the shortest path. The travel time is calculated by dividing the distance by the velocity and approximately follows a (truncated) normal distribution [25]. To estimate the travel time distribution of an arc, the distributions of multiple road segments are considered.

Let  $X^a(t)$  be the random variable of travel time at time  $t$  on arc  $a$ . Suppose that the arc consists of  $m$  road segments and let  $X_1^a(t), X_2^a(t), \dots, X_m^a(t)$  be the random variables of travel time at time  $t$  on the road segments  $i = 1, 2, \dots, m$ , respectively. It is assumed that each random variable for the travel time on a road segment approximately follows a normal distribution,  $X_i^a(t) \sim N(\mu_{x_i}, \sigma_{x_i}^2)$ ,  $i = 1, \dots, m$ . Since the travel time of each road segment is calculated by dividing the distance by the velocity, the random variable of travel time for an arc is the sum of random variables of the travel times of the road segments. That is,

$$X^a(t) = X_1^a(t) + X_2^a(t) + \dots + X_m^a(t).$$

Assuming that the travel times of the road segments are independent of each other, the mean and variance of the travel time

for arc  $a$  can be defined as follows:

$$\begin{aligned}\mu_{X^a(t)} &= E[X^a(t)] \\ &= E[X_1^a(t)] + E[X_2^a(t)] + \dots + E[X_m^a(t)] \\ \sigma_{X^a(t)}^2 &= \text{Var}[X^a(t)] \\ &= \text{Var}[X_1^a(t)] + \text{Var}[X_2^a(t)] + \dots + \text{Var}[X_m^a(t)].\end{aligned}$$

We refer readers to an appendix for the detailed discussion of independence and normality assumptions of travel times. Using the histogram approximation, we can discretize the travel time of the arc as follows:

$$\begin{aligned}P(X^a(t) = x) &= P(x - 0.5 \leq X^a(t) \leq x + 0.5) \\ &= P\left(\frac{x - 0.5 - \mu_{X^a(t)}}{\sigma_{X^a(t)}} \leq \frac{V(t) - \mu_{X^a(t)}}{\sigma_{X^a(t)}}\right. \\ &\quad \left. \leq \frac{x + 0.5 - \mu_{X^a(t)}}{\sigma_{X^a(t)}}\right) \\ &= \Phi\left(\frac{x + 0.5 - \mu_{X^a(t)}}{\sigma_{X^a(t)}}\right) - \Phi\left(\frac{x - 0.5 - \mu_{X^a(t)}}{\sigma_{X^a(t)}}\right).\end{aligned}$$

In reality, the datum of the speed from sensors on the road is a value at a moment, not over a period. Thus, to use the data, we calculate the average of the values collected during a period. The data from several sensors on a road are collected to calculate the average speed on that road. The averages of the speeds of road segments are used to calculate the average speed of the arc between two nodes.

## V. ROLLOUT POLICY

In this section, we present the approach taken by the solution to the DVRP under consideration. In approximate dynamic programming (ADP), we use  $\tilde{v}(n', t', \vec{s}'_t, M'_t)$  instead of the exact value,  $v^*(n', t', \vec{s}'_t, M'_t)$  and thereby obtain a suboptimal solution. To avoid the *curse of dimensionality*, the underlying principle of the ADP approach is that a ‘good’ solution instead of the ‘optimal’ one can be sufficient. We employ the rollout algorithm which is a policy iteration procedure for forward dynamic programming based on a multi-step look-ahead process, assuming a given base policy (heuristic). A decision heuristic rule is defined to evaluate the cost-to-go function in the rollout algorithm. The rollout algorithm solves the problem iteratively using the Bellman equation and heuristic rules, and makes step-by-step decisions about where the vehicle should go next, given the uncertainty of traffic conditions, until all customers are serviced. As a neuro-dynamic programming method, the rollout policy often uses a Monte Carlo simulation to generate scenarios (scenarios of possible changes in traffic conditions in this case) which can significantly reduce computational burden.

For implementation, a one-step look-ahead rollout approach is commonly used, which is similar to a rolling horizon approach. Fig. 2 illustrates the framework of the one-step rollout algorithm. Let  $H(\Omega_t)$  be the decision rule or heuristic for evaluating the cost-to-go function. When a vehicle is at state  $\Omega_t$ , for a given policy  $\pi(\Omega_t)$ , we consider all the possible next states  $\Omega_{t+1}$  and execute the heuristic for each. Once the values

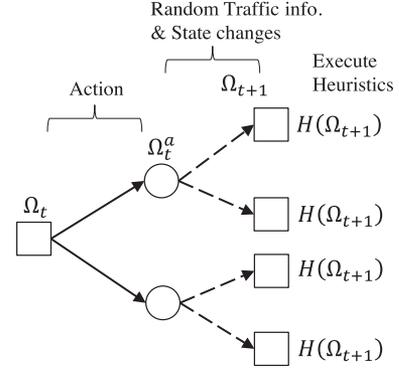


Fig. 2. One-step rollout algorithm framework.

of all the next states have been evaluated using the heuristic, the best policy is obtained using Equation (5) below. We formalize the rollout algorithm as follows:

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### Rollout algorithm

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#### 1. Initialization

Current state  $\Omega_t$ , let  $t = 0$

#### 2. While current state $\Omega_t \in \Omega$ is not terminal state Do

#### 3. For all possible next states $\Omega_{t+1}$ (one step ahead) Do

$$\pi^*(\Omega_t) = \arg \min_{\pi(\Omega_t) \in \Psi} \left\{ g(\Omega_t, \pi(\Omega_t)) + \sum_{\forall \Omega_{t+1}} P(\Omega_{t+1} | \Omega_t, \pi(\Omega_t)) v^{\pi_{H(\Omega_{t+1})}}(\Omega_{t+1}) \right\}. \quad (5)$$

#### 4. Store $\pi^*(\Omega_t)$

#### 5. Update current state by $\Omega_{t+1}$ with $\pi^*(\Omega_t)$ and a realization of $\vec{s}'_t$

#### 6. $t \leftarrow t + 1$

---

First, the algorithm initializes the current state in Step 1. Step 2 starts a loop in which the procedure for each successive state is conducted iteratively until the terminal state is reached. In Step 3, a loop over all possible next states is executed to find the policy which gives the minimum expected travel cost. The immediate cost  $g(\Omega_t, \pi(\Omega_t))$  of the policy is calculated in the loop. The algorithm executes the heuristic  $H(\Omega_{t+1})$  to generate the heuristic policy  $\pi_{H(\Omega_{t+1})}$  and calculate the expected future cost. Then, using that policy, the best policy is calculated. Step 4 stores the best policy for the state. In Step 5, the current state is updated from a realization (observation) of states and the next cycle of the loop set started at Step 2 is undertaken to determine the next destination. Equation (5) modifies Equation (2) by substituting an approximated value,  $\tilde{v}(\Omega_{t'}) \equiv v^{\pi_{H(\Omega_{t'})}}(\Omega_{t'})$  instead of  $v^*(\Omega_{t'})$ .

To run the rollout algorithm, we need to use a base policy (heuristic) to approximate the cost-to-go function. Although any heuristic can be used if it guarantees the termination of the rollout algorithm, unless it is sequentially inconsistent in which case the graph becomes acyclic [3], we use the simple and fast

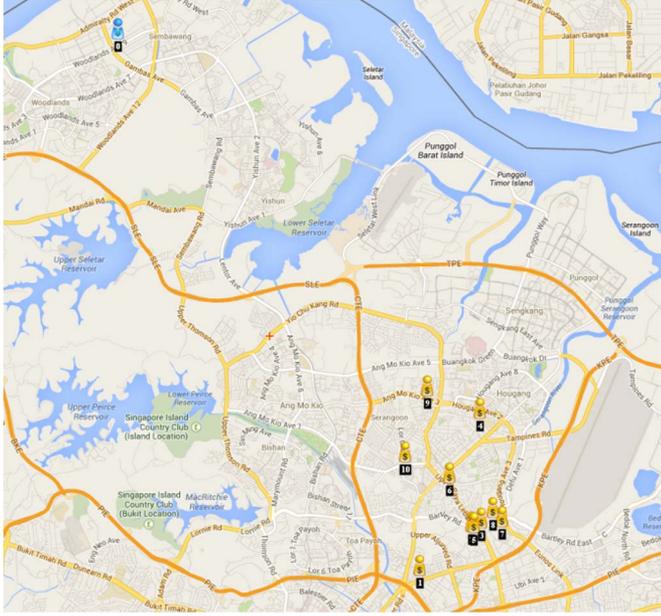


Fig. 3. A Singapore delivery company's customer cluster comprised of 1 depot and 10 customers.

nearest neighborhood search method commonly used with the travelling salesman problem. When we calculate  $v^{\pi_H(\Omega_{t'})}(\Omega_{t'})$ , there are two options. One is to calculate the exact values recursively, which still engenders a computational burden, and the other to use a Monte Carlo simulation with possible scenarios. Bertsekas [2] introduced the simulation method to calculate good approximations of the cost-go-function, and Novoa and Storer [32] have adopted the simulation method as well as the exact method. In this paper, we use the Monte Carlo simulation method, and generate a finite number of scenarios from the one-step forward state  $\Omega_{t+1}$  to the termination state to estimate the future expected cost-go-function.

## VI. CASE STUDY AND NUMERICAL EXPERIMENTS

We now present our approach to applying the MDP model as well as estimations of traffic congestion dynamics and travel time distributions to the problem faced by a logistics/delivery company in Singapore.

### A. Illustration and Applications of the Proposed Approach

To illustrate, we consider one customer cluster shown in Fig. 3, which consists of a single depot and ten customers. The depot is located on the northwest part of the map while all the customers are located on the southeast part. The detailed locations of the eleven nodes are given in Table II.

There are 110 different arcs with eleven nodes in graph  $G$ . The shortest path corresponding to each arc connecting two nodes for the eleven nodes was derived using Google API. For example, the shortest path from the depot to customer 1 consists of seven road segments such as *Woodland Link*, *Woodland Loop*, *Woodland Avenue*, *Gambas Avenue*, *Upper Serangoon Road*, *Macpherson Road* and *Genting Lane*. After applying the Google API to every arc, 28 different road segments for the 110 arcs were identified as shown in Table III.

TABLE II  
LOCATIONS OF CUSTOMER NODES IN THE CLUSTER IN FIG. 3

Nodes	Postal code	Latitude	Longitude
<i>Depot</i>	73****	1.4*****	103.8***
<i>Customer 1</i>	34****	1.3*****	103.8***
<i>Customer 2</i>	36****	1.3*****	103.8***
<i>Customer 3</i>	53****	1.3*****	103.8***
<i>Customer 4</i>	53****	1.3*****	103.8***
<i>Customer 5</i>	53****	1.3*****	103.8***
<i>Customer 6</i>	53****	1.3*****	103.8***
<i>Customer 7</i>	53****	1.3*****	103.8***
<i>Customer 8</i>	53****	1.3*****	103.8***
<i>Customer 9</i>	54****	1.3*****	103.8***
<i>Customer 10</i>	55****	1.3*****	103.8***

TABLE III  
28 DIFFERENT ROAD SEGMENTS FOR THE 110 ARCS

	Road Name		Road Name
1	AIRPORT ROAD	15	PEREIRA ROAD
2	ANG MO KIO AVENUE	16	POH HUAT ROAD
3	BARTLEY ROAD	17	SERANGOON AVENUE
4	BARTLEY ROAD EAST	18	TAI SENG AVENUE
5	BOUNDARY ROAD	19	TAI SENG DRIVE
6	BRADDELL ROAD	20	TAI SENG STREET
7	GAMBAS AVENUE	21	TAMPINES ROAD
8	GENTING LN	22	UPPER PAYA LEBAR ROAD
9	GENTING LINK	23	UPPER SERANGOON ROAD
10	HOUGANG AVENUE	24	WOODLANDS AVENUE
11	KIM CHUAN DRIVE	25	WOODLANDS LINK
12	KIM CHUAN ROAD	26	WOODLANDS LOOP
13	LORONG AH SOO ROAD	27	YIO CHU KANG LINK
14	MACPHERSON ROAD	28	YIO CHU KANG ROAD

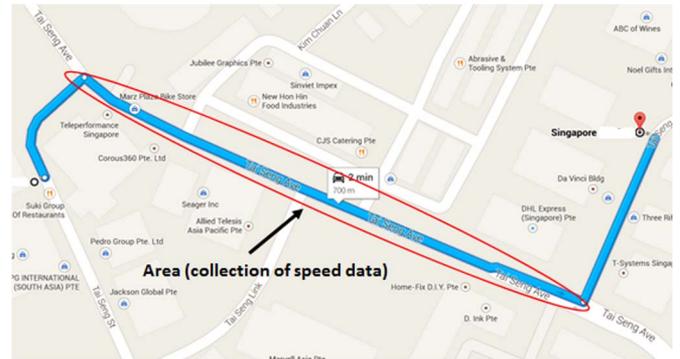


Fig. 4. Road segment of *Tai Seng Avenue*.

The traffic data collected from the Land Transport Authority (LTA)<sup>1</sup> in Singapore are utilized to measure the speed of vehicles which is eventually used to estimate the dynamics of congestion status as well as the travel time distribution of each road segment in Table III.

For example, let us consider the arc which physically corresponds to the shortest path, which is shown in Fig. 4. Fourteen

<sup>1</sup><http://www.lta.gov.sg/>

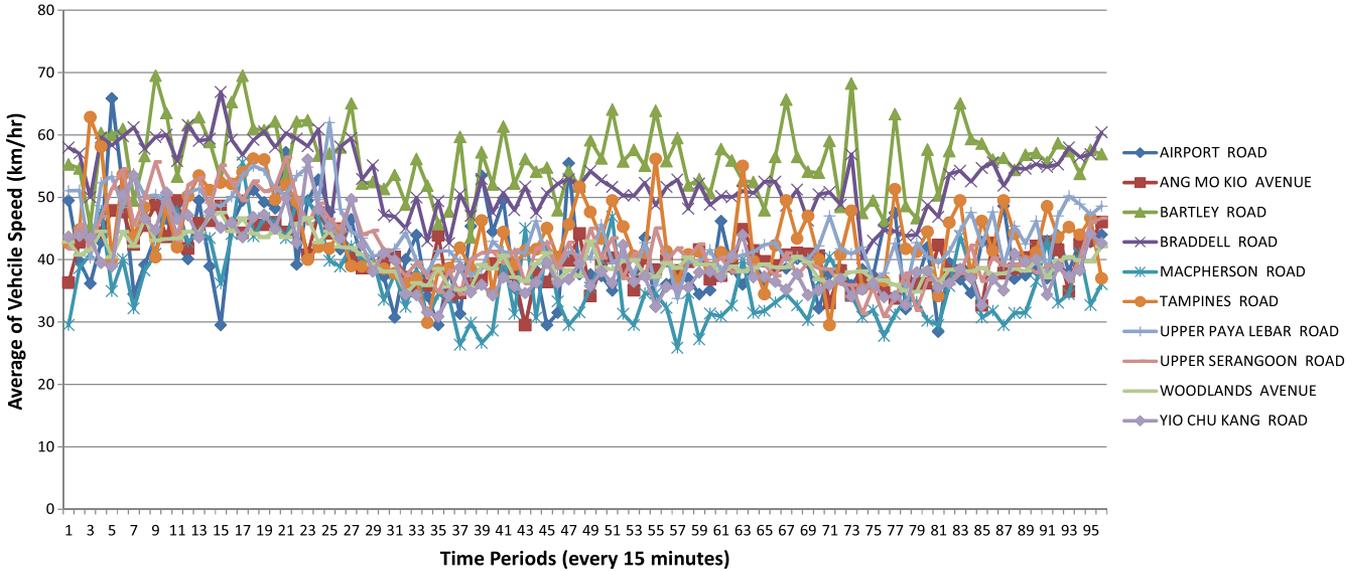


Fig. 5. Average vehicle speeds on particular roads throughout the day (period 1 starts at 12 am).

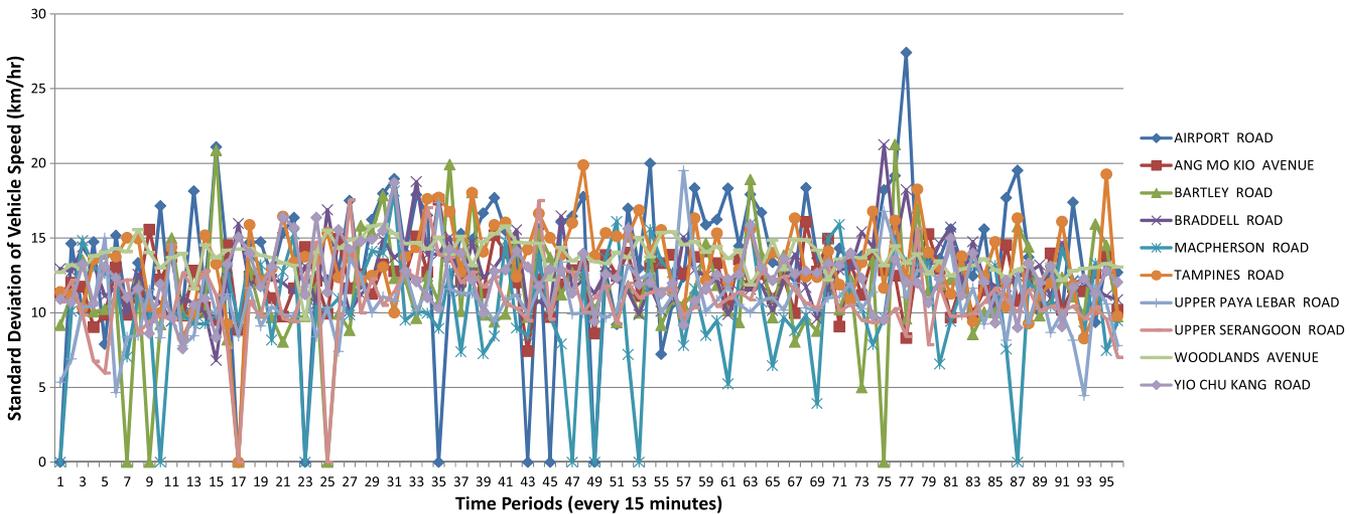


Fig. 6. Standard deviations of vehicle speeds on particular roads throughout the day.

vehicle speed sensors are currently installed along the road segments making up the shortest path, and the data collected from the ten sensors along the road segment *Tai Seng Avenue* can be utilized. To estimate the vehicle speed distributions of the 28 road segments in Table III, we identified the associated sensors and compiled the speed data for each road segment from the traffic data provided by the LTA. For further analysis, the speed data covering 24 hours was divided into 96 time periods (i.e., 15-minute intervals). We calculated the means and variances for every interval between July 14 and July 21, 2014 (8 days), and the resulting means and standard deviations for some of the 28 roads are presented in Figs. 5 and 6. Fig. 5 shows that, except for a few road segments, the average speeds of vehicles along those of interest are stable. However, Fig. 6 shows that the variability of vehicle speed is larger during the night.

With the speed information compiled for 28 different road segments, we can estimate the dynamics of traffic congestion states and the travel time distributions of 110 arcs, based on

the approach outlined in Section IV. When considering the dynamics of the traffic congestion states of an arc, we assign one of two congestion levels, *congested* and *uncongested*, with *congested* corresponding to a velocity of less than 40 km/h.

The averages and standard deviations of the estimated travel time distributions are presented in Figs. 7 and 8 respectively. As shown in Fig. 7, when the arcs between the depot and the clustered customers are long, their average travel times range from 25 to 42 minutes. On the other hand, when any two nodes are close each other, the average travel times range from 1 to 12 minutes. As illustrated in Fig. 8, the standard deviations of arc travel times range from 1 to 8 minutes.

Using the estimations of traffic congestion state transition probabilities and arc travel time distributions as input data, we solve the dynamic vehicle routing problem of the company with the approach based on the rollout policy discussed in Section V. The nearest neighborhood heuristic is used as a base policy, and a Monte Carlo simulation generates 200 scenarios

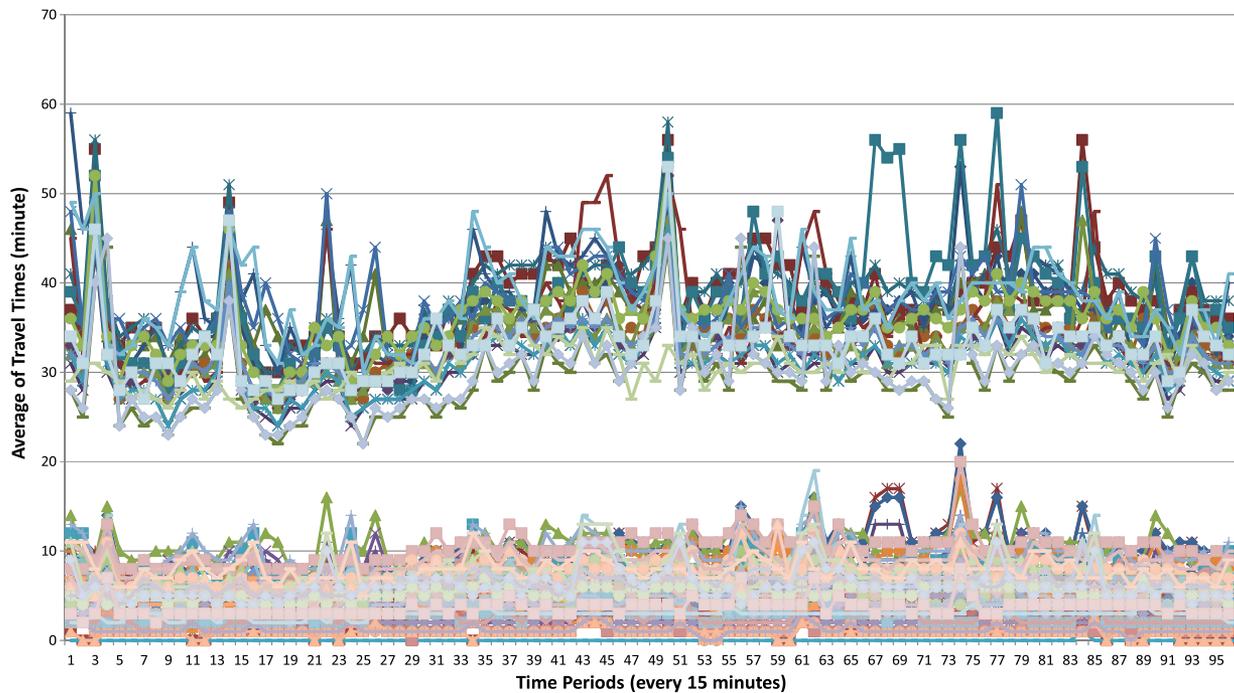


Fig. 7. Average of travel times for 110 arcs.

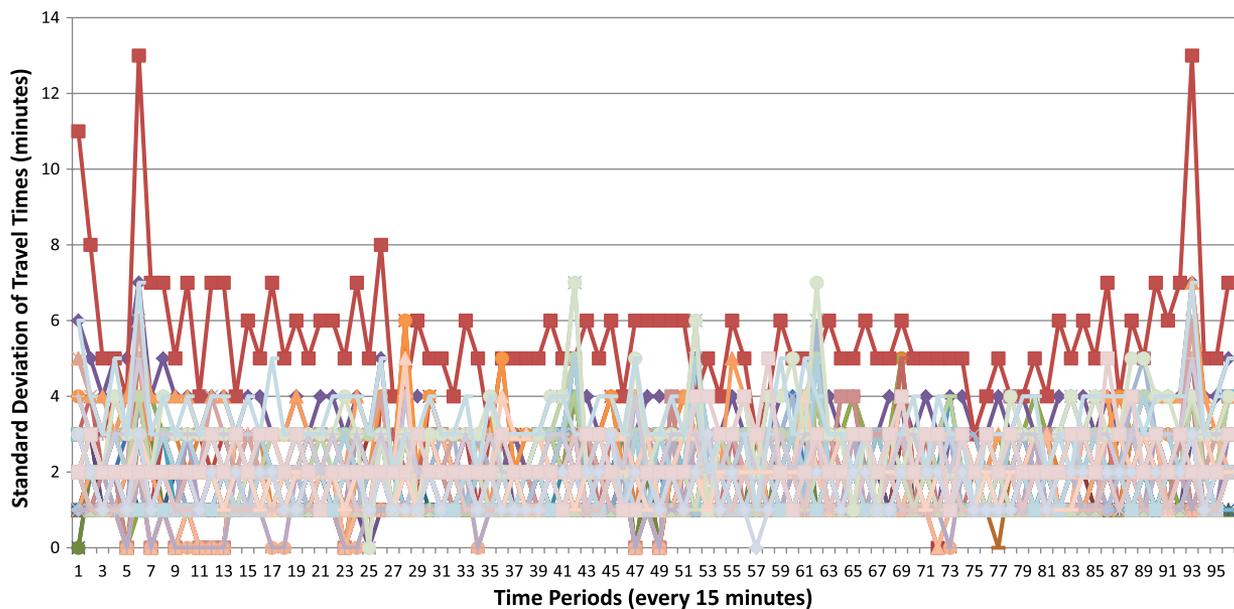


Fig. 8. Standard deviation of travel times for 110 arcs.

to approximate cost-go-functions at every decision epoch. Table IV presents the results of applying the rollout policy to the customer cluster above.

As Table IV indicates, as the variability of the travel time increases, the average and standard deviations of the total expected travel costs for a given policy derived by the proposed approach also increase. Next, we further evaluate the performance of the proposed approach through a comparison with the performance achieved by the current practices of the Singapore delivery company.

### B. Comparison With the Current Practice by the Delivery Company in Singapore

We now present the performance of the proposed approach on the delivery network in Singapore, and highlight the potential savings from exploiting historical and real-time traffic congestion information. The company delivers commodities to 100 customers widely dispersed around Singapore as shown in Fig. 9.

The depot is located at the same spot as one cluster example in Fig. 3. Customers demand different quantities, and their demands are known to the company in advance. The company

TABLE IV  
SOLUTIONS OF DVRP FOR ONE CLUSTER EXAMPLE

Standard deviation of travel time	Total expected travel costs for the route derived from the proposed approach	
	Mean	Standard deviation
$\sigma * 0.5$	106.9463	13.45635
$\sigma$	114.1013	22.68664
$\sigma * 1.5$	130.527	28.20614
$\sigma * 2$	123.9754	33.70702



Fig. 9. Singapore map showing the customers of the logistics/delivery company for case study.

divides its 100 customers by postal code into 21 customer clusters as presented in Table V, and to deliver commodities to all its customers, it assigns one truck to each cluster. Thus, company currently operates 21 trucks. The trucks have sufficient capacity to cover all the demand in a cluster.

Of the customer clusters in Table V, we perform numerical experiments with five which contain more than seven customers (clusters 1, 2, 5, 13, and 19) for performance evaluation purposes. The company's current practice is that, for each customer cluster, the truck is driven along a pre-determined route determined by a static vehicle routing problem that simply minimizes the total travel distance. Thus, while our approach takes them into account, the current practice of the company ignores the effect of traffic congestion state changes over time. For each customer cluster, we apply the proposed approach to determine the dynamic route. For the experiments, we use the travel time distributions in Section VI-A, generate 30 scenarios to simulate the traffic dynamics and have the vehicles start their deliveries at 10 am. We note that generating scenarios from the estimated system dynamics and distributions and evaluating the performance of the proposed approach based on the scenarios are not new in literature (e.g., [20] or [25]). The results for the five customer clusters examined are presented in Fig. 10, and Table VI compares the total travel costs for static and dynamic routes. We remark that the proposed algorithm was

TABLE V  
CUSTOMER CLUSTERS OF THE COMPANY

Clusters	Customers
1	1,2,3,4,5,6,7,8,9,10
2	11,12,13,14,15,16,17,18,19,20,21
3	22,23,24,25,26
4	27
5	28,29,30,31,32,33,34,35,36,37
6	38,39,40,41,42
7	43,44,45,46,47,48
8	49,50
9	51,52,53,54,55
10	56,57
11	58,59,60,61
12	62,63,64,65,66
13	67,68,69,70,71,72,73
14	74
15	75,76,77,78
16	79,80
17	81,82,83
18	84,85
19	86,87,88,89,90,91,92,93,94
20	95,96
21	97,98,99,100

implemented in Microsoft C# and run on a PC with an Intel Xeon 3.2-GHz processor and 16 GB of random access memory. Furthermore, we were able to obtain the policy of every cluster in under 2 seconds.

The numerical experiments clearly indicate that the dynamic routes derived by the proposed approach outperform the static ones currently used. As Goodson *et al.* [19] described, trucking companies are interested in making even 1% or 2% gains in routing efficiency from new methods to improve their operating margins. However, according to the numerical results above, a 7% improvement in total travel time can potentially be achieved when the company utilizes the rollout-based DVRP algorithms which take into account traffic congestion.

## VII. CONCLUSION

This paper focuses on the DVRP with stochastic and dynamic travel times, adopts a rollout algorithm based approach to the solution, and proposes a realistic way to estimate traffic congestion state transition probabilities and the travel time distributions of arcs in the transportation network. The Euclidean distance between two nodes differs too widely from the distance vehicles practically travel on the road. The arc between a pair of nodes actually consists of multiple road segments which have different traffic distributions. Thus, we have to consider the travel time distributions of road segments when estimating the travel time distributions of network arcs.

This paper proposes a realistic estimation method for the travel time distribution of each arc considering road segment distributions. Compared to demand dynamics, the DVRP with travel time dynamics is more complicated since in addition to traffic congestion states, the number of arcs is larger than the

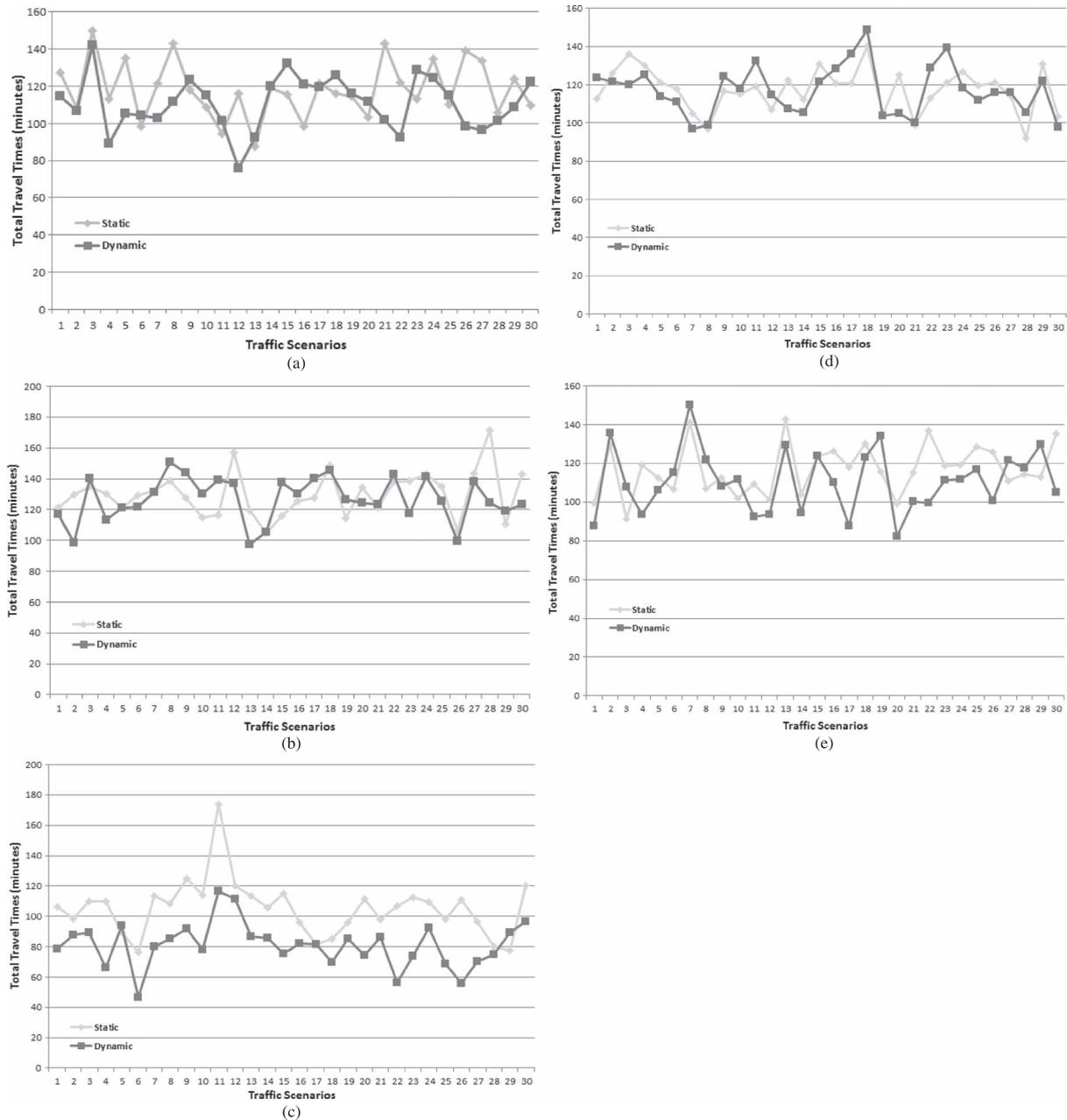


Fig. 10. Comparison of total travel times (costs) for static and dynamic routes in clusters 1, 2, 5, 13 and 19. (a) Cluster 1. (b) Cluster 2. (c) Cluster 5. (d) Cluster 13. (e) Cluster 19.

TABLE VI  
COMPARISONS OF TOTAL TRAVEL COSTS BETWEEN  
DYNAMIC AND STATIC SOLUTIONS

	Total Travel Costs	
	Static	Dynamic
Cluster 1	118.2023	110.7255
Cluster 2	129.822	126.8941
Cluster 5	105.3253	81.01403
Cluster 13	117.429	117.0881
Cluster 19	116.9189	110.7877
Total	587.6976	546.5095
Improvement	7%	

number of nodes. Thus, solving this problem on large scale is likely to be intractable. For this reason, to avoid the curse of dimensionality, we suggested a solution approach based on a rollout algorithm with a Monte Carlo simulation. We also compared the solutions with the current practices performed by the company of interest. The results show that an improvement of 7% can potentially be achieved when the rollout policy for a DVRP with traffic congestion is analyzed.

We consider the shortest path between a pair of nodes when estimating arcs' travel time distributions. In order to be exact, the shortest path should be derived by solving the stochastic shortest path problem. Thus, the integration of the stochastic shortest path problem into the dynamic vehicle routing problem

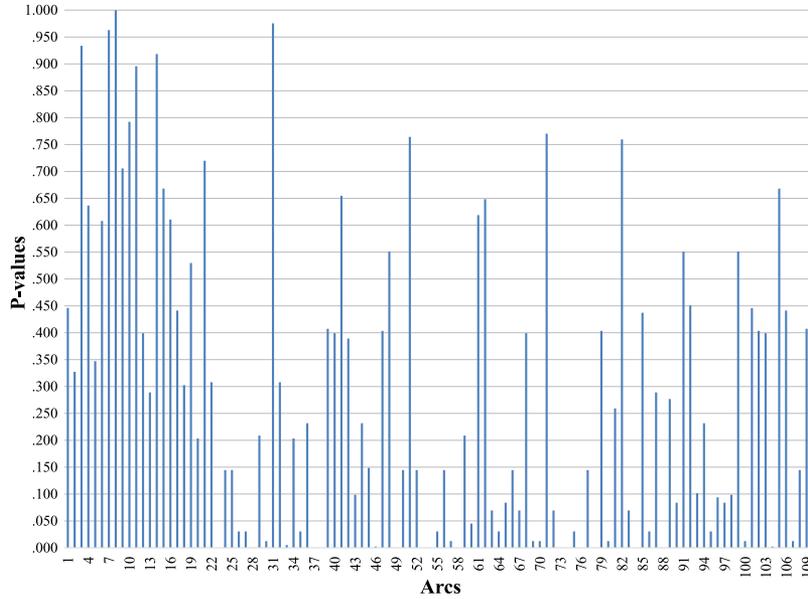


Fig. 11. Test results of normality using Kolmogorov-Smirnov test.

is a future challenge. The stochastic and dynamic VRP incorporating both demand and travel time dynamics would also be an interesting focus for future research.

#### APPENDIX INDEPENDENCE AND NORMALITY ASSUMPTIONS OF TRAVEL TIMES AMONG ARCS

We now discuss the plausibility of the independence and normality assumptions of travel times among arcs in the graph  $G$  in Section IV. First, it is assumed that travel times of road segments are independent. In the real world, there may exist correlations among travel times and some research indeed takes into account the aspects. For example, Figliozzi [12] considered all the correlations between two adjacent link travel times in a path when they calculated the standard deviation of the link travel time. However, they considered the correlations among the links in a given route, not in the whole network, to investigate the impact of congestion on duration constrained tours. We remark that each vehicle in the problem under consideration is supposed to make a decision where to go next at every customer location while monitoring the whole traffic congestion status on the road network—in other words, the tour which the truck drivers may follow is unknown in advance before (s)he reaches the final stage. In this case, we have to consider all the correlations among observable roads or road segments in the whole road network if we need to do so.

However, as Change *et al.* [6] pointed out, the independence assumption is not that unrealistic, and according to Lo and Tung [28], the assumption seems to be reasonable—especially, for relatively minor network disruptions such as traffic incidents and parking violations (as like Singapore). It should be also mentioned that this assumption has also been frequently used in the literature [33], [34]. In fact, Park [35] investigated the correlation between neighboring links and showed that the

correlation between neighboring links generally decreases over time. Moreover, it is not hard to find out research that assumes that traffic congestion dynamics are governed by a discrete-time Markov chain under the independence assumption of arcs' congestion states, as we also assume in this study (e.g., [20] or [25]). Specifically, Kim *et al.* [25] discussed the details of data collection (see [25, Sec. III-A]) and the construction of Markov chain [ $P(s(t+1)|s(t))$ ] (see [25, Sec. III-B]). We can observe the similar approach in Guner *et al.* [20] as well.

Next, we also assume that travel times are normally distributed. As presented in Section II, travel times are often assumed to follow normal distributions in literature. Although the confirmation has not been presented in all, the normality assumption has been supported by three different ways such as the *central limit theorem*, *simulation*, or *goodness-of-fit test*. By the central limit theorem, when the number of nodes gets large enough, one can assume that the travel time is normally distributed. Another approach to confirm the assumption of normal distribution is to use a simulation. Using parameters of data collected for speed or distance, one may generate the travel times and check whether the normal distribution is well fitted to the resulting random samples. In this paper, we have conducted the Kolmogorov-Smirnov goodness-of-fit test with our available data set.

Fig. 11 shows the results from testing for normality with Kolmogorov-Smirnov test. In the figure, the results indicate that most p-values are more than 0.05, which implies that the majority of the travel times of arcs are normally distributed. Thus, the results support that the normality assumption holds for most arcs on the road network under study.

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## REFERENCES

- [1] F. Azadian, A. E. Murat, and R. B. Chinnam, "Dynamic routing of time-sensitive air cargo using real-time information," *Transp. Res. E, Logist. Transp. Rev.*, vol. 48, no. 1, pp. 355–372, Jan. 2012.
- [2] D. P. Bertsekas, *Dynamic Programming and Optimal Control*, vol. 2. Belmont, MA, USA: Athena Scientific, 2001.
- [3] D. P. Bertsekas, "Rollout algorithms for discrete optimization: A survey," in *Handbook of Combinatorial Optimization*, P. M. Pardalos, D. Z. Du, and R. L. Graham, Eds. New York, NY, USA: Springer-Verlag, 2013, pp. 2989–3013.
- [4] D. P. Bertsekas and D. A. Castanon, "Rollout algorithms for stochastic scheduling problems," *J. Heuristics*, vol. 5, no. 1, pp. 89–108, Apr. 1999.
- [5] D. P. Bertsekas and J. N. Tsitsiklis, "Neuro-dynamic programming: An overview," in *Proc. 34th IEEE Conf. Decision Control*, vol. 1–4, 1995, pp. 560–564.
- [6] T.-S. Chang, Y.-W. Wan, and W. T. OOI, "A stochastic dynamic traveling salesman problem with hard time windows," *Eur. J. Oper. Res.*, vol. 198, no. 3, pp. 748–759, Sep. 2009.
- [7] T. Cheong and C. C. White, "Dynamic traveling salesman problem: Value of real-time traffic information," *IEEE Trans. Intell. Transp. Syst.*, vol. 13, no. 2, pp. 619–630, Jun. 2012.
- [8] M. Ciavotta, C. Meloni, and M. Pranzo, "Scheduling dispensing and counting in secondary pharmaceutical manufacturing," *AICChE J.*, vol. 55, no. 5, pp. 1161–1170, May 2009.
- [9] K. Cooke and E. Halsey, "The shortest route through a network with time-dependent internodal transit times," *J. Math. Anal. Appl.*, vol. 14, no. 3, pp. 493–498, Jun. 1966.
- [10] B. C. Csaji and L. Monostori, "Adaptive stochastic resource control: A machine learning approach," *J. Artif. Intel. Res.*, vol. 32, pp. 453–486, 2008.
- [11] J. W. Ding, C. F. Wang, F. H. Meng, and T. Y. Wu, "Real-time vehicle route guidance using vehicle-to-vehicle communication," *IET Commun.*, vol. 4, no. 7, pp. 870–883, Apr. 2010.
- [12] M. A. Figliozzi, "The impacts of congestion on commercial vehicle tour characteristics and costs," *Transp. Res. E, Logist. Transp. Rev.*, vol. 46, no. 4, pp. 496–506, Jul. 2010.
- [13] M. A. Figliozzi, "The impacts of congestion on time-definitive urban freight distribution networks CO2 emission levels: Results from a case study in Portland, Oregon," *Transp. Res. C, Emerging Technol.*, vol. 19, no. 5, pp. 766–778, Aug. 2011.
- [14] M. A. Figliozzi, "The time dependent vehicle routing problem with time windows: Benchmark problems, an efficient solution algorithm, and solution characteristics," *Transp. Res. E, Transp. Rev.*, vol. 48, no. 3, pp. 616–636, May 2012.
- [15] B. Fleischmann, S. Gnutzmann, and E. Sandvoss, "Dynamic vehicle routing based on online traffic information," *Transp. Sci.*, vol. 38, no. 4, pp. 420–33, Nov. 2004.
- [16] H. Frank, "Shortest paths in probabilistic graphs," *Oper. Res.*, vol. 17, no. 4, pp. 583–599, Jul./Aug. 1969.
- [17] L. P. Fu and L. R. Rilett, "Expected shortest paths in dynamic and stochastic traffic networks," *Transp. Res. B, Methodol.*, vol. 32, no. 7, pp. 499–516, Sep. 1998.
- [18] S. Gao and H. Huang, "Real-time traveler information for optimal adaptive routing in stochastic time-dependent networks," *Transp. Res. C, Emerging Technol.*, vol. 21, no. 1, pp. 196–213, Apr. 2012.
- [19] J. C. Goodson, J. W. Ohlmann, and B. W. Thomas, "Rollout policies for dynamic solutions to the multivehicle routing problem with stochastic demand and duration limits," *Oper. Res.*, vol. 61, no. 1, pp. 138–154, Jan./Feb. 2013.
- [20] A. R. Guner, A. Murat, and R. B. Chinnam, "Dynamic routing under recurrent and non-recurrent congestion using real-time ITS information," *Comput. Oper. Res.*, vol. 39, no. 2, pp. 358–373, Feb. 2012.
- [21] A. Gupta, V. Nagarajan, and R. Ravi, "Approximation algorithms for VRP with stochastic demands," *Oper. Res.*, vol. 60, pp. 123–127, Jan./Feb. 2012.
- [22] A. Haghani and S. Jung, "A dynamic vehicle routing problem with time-dependent travel times," *Comput. Oper. Res.*, vol. 32, no. 11, pp. 2959–2986, Nov. 2005.
- [23] R. W. Hall, "The fastest path through a network with random time-dependent travel-times," *Transp. Sci.*, vol. 20, no. 3, pp. 182–188, Aug. 1986.
- [24] H. Julia, M. Dessouky, and P. A. Ioannou, "Truck route planning in non-stationary stochastic networks with time windows at customer locations," *IEEE Trans. Intell. Transp. Syst.*, vol. 7, no. 1, pp. 51–62, Mar. 2006.
- [25] S. Kim, M. E. Lewis, and C. C. White, "Optimal vehicle routing with real-time traffic information," *IEEE Trans. Intell. Transp. Syst.*, vol. 6, no. 2, pp. 178–188, Jun. 2005.
- [26] H. N. Koutsopoulos and H. Xu, "An information discounting routing strategy for advanced traveler information systems," *Transp. Res. C, Emerging Technol.*, vol. 1, no. 3, pp. 249–264, Sep. 1993.
- [27] C. Lee, K. Lee, and S. Park, "Robust vehicle routing problem with deadlines and travel time/demand uncertainty," *J. Oper. Res. Soc.*, vol. 63, no. 9, pp. 1294–1306, Sep. 2012.
- [28] H. K. Lo and Y.-K. Tung, "Network with degradable links: Capacity analysis and design," *Transp. Res. B, Methodol.*, vol. 37, no. 4, pp. 345–363, May 2003.
- [29] S. Lorini, J. Y. Potvin, and N. Zufferey, "Online vehicle routing and scheduling with dynamic travel times," *Comput. Oper. Res.*, vol. 38, no. 7, pp. 1086–1090, Jul. 2011.
- [30] Y. Nakamura, E. Taniguchi, T. Yamada, and N. Ando, "Selecting a dynamic and stochastic path method for vehicle routing and scheduling problems," *Procedia Soc. Behav. Sci.*, vol. 2, pp. 6042–6052, 2010.
- [31] L. R. Nielsen, K. A. Andersen, and D. Pretolani, "Integrating stochastic time-dependent travel speed in solution methods for the dynamic dial-a-ride problem," *Eur. J. Oper. Res.*, vol. 238, no. 11, pp. 18–30, Oct. 2014.
- [32] C. Novoa and R. Storer, "An approximate dynamic programming approach for the vehicle routing problem with stochastic demands," *Eur. J. Oper. Res.*, vol. 196, no. 2, pp. 509–515, Jul. 16 2009.
- [33] M. Ng, W. Y. Szeto, and S. T. Waller, "Distribution-free travel time reliability assessment with probability inequalities," *Transp. Res. B, Methodol.*, vol. 45, no. 6, pp. 852–866, Jul. 2011.
- [34] M. Ng and S. T. Waller, "A computationally efficient methodology to characterize travel time reliability using the fast Fourier transform," *Transp. Res. B, Methodol.*, vol. 44, no. 10, pp. 1202–1219, Dec. 2010.
- [35] D. Park, "Forecasting freeway link travel times with a multilayer feed-forward neural network," *Comput.-Aided Civil Infrastruct. Eng.*, vol. 14, no. 5, pp. 357–367, Sep. 1999.
- [36] W. B. Powell, *Approximate Dynamic Programming Solving the Curses of Dimensionality*, 2nd ed. Hoboken, NJ, USA: Wiley, 2011.
- [37] M. Schilde, K. F. Doerner, and R. F. Hartl, "Integrating stochastic time-dependent travel speed in solution methods for the dynamic dial-a-ride problem," *Eur. J. Oper. Res.*, vol. 238, no. 11, pp. 18–30, Oct. 2014.
- [38] N. Secomandi, "A rollout policy for the vehicle routing problem with stochastic demands," *Oper. Res.*, vol. 49, no. 5, pp. 796–802, Sep./Oct. 2001.
- [39] N. Secomandi, "Analysis of a rollout approach to sequencing problems with stochastic routing applications," *J. Heuristics*, vol. 9, no. 4, pp. 321–352, Sep. 2003.
- [40] E. Taniguchi and H. Shimamoto, "Intelligent transportation system based dynamic vehicle routing and scheduling with variable travel times," *Transp. Res. C, Emerging Technol.*, vol. 12, no. 3/4, pp. 235–250, Jun./Aug. 2004.
- [41] D. Tas, N. Dellaert, T. van Woensel, and T. de Kok, "Vehicle routing problem with stochastic travel times including soft time windows and service costs," *Comput. Oper. Res.*, vol. 40, no. 1, pp. 214–224, Jan. 2013.
- [42] D. Tas, N. Dellaert, T. van Woensel, and T. de Kok, "The time-dependent vehicle routing problem with soft time windows and stochastic travel times," *Transp. Res. C, Emerging Technol.*, vol. 48, pp. 66–83, Nov. 2014.
- [43] J. K. Wang, N. Indra-Payoong, A. Sumalee, and S. Panwai, "Vehicle reidentification with self-adaptive time windows for real-time travel time estimation," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 2, pp. 540–552, Apr. 2014.



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